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ADVANCED MATHEMATICAL RESEARCH

AF-AFOSR-359-66

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Felix R. Albrecht
Michael F. Atiyah
Jonathan P. Brezin
Lothar Gerritzen
Jerome A. Goldstein
Hervé Jacquet
Yakar I. Kannai
Wilhelm Klingenberg
Kiyosato Okamoto
John V. Ryff
Paul J. Sally, Jr.
Shunichi Tanaka

Manuscripts submitted:

Felix R. Albrecht, Control vector fields on manifolds and attainability.

Michael F. Atiyah, Algebraic topology and operators in Hilbert space.

————————, The signature of fibre-bundles.

Jerome A. Goldstein, Abstract Cauchy problems.

————————, A lemma on the generation of one parameter semi-groups of operators.

————————, On a connection between first and second order differential equations in Banach spaces.

————————, Second order Ito processes.

————————, Some remarks on infinitesimal generators of analytic semi-groups.

————————, Time dependent hyperbolic equations.

Yakar I. Kannai, On the asymptotic behavior of resolvent kernels, spectral functions and eigenvalues of semi-elliptic systems.

Wilhelm Klingenberg, Closed geodesics.

————————, Simple closed geodesics on a Riemannian manifold homeomorphic to the sphere.

————————, The space of closed curves on a projective space.

John V. Ryff, Majorized functions and measures.

Paul J. Sally, Jr. (with J. A. Shalika), Characters of the discrete series of representations of SL(2) over a local field.

Manuscripts continued:

Shunichi Tanaka (with J. A. Shalika), On an explicit construction of a certain class or automorphic forms.

ADDENDUM

Technical Report for 1966-68

Grant No. AF-AFOSR-359-66

Project Task: 9749-02

Author: Felix R. Albrecht

I have continued to investigate the geometric properties of sets of attainability of control vector fields on differentiable manifolds. Some progress has been made in applying previously developed methods to the problem of existence of geodesics joining two given points on a manifold.

Author: Michael F. Atiyah

My main concern has been the study of families of elliptic operators. The general index theory is being developed jointly with I. M. Singer. In addition, an interesting connection with the signature of fibre-bundles has come to light.

Author: Jonathan P. Brezin

I continued to work on unitary representation theory for solvable Lie groups. The greater part of my work has been to find ways of realizing p-adic linear groups as groups of operators. Partial results will probably appear in a joint paper with Paul J. Sally.

Author: Lothar Gerritzen

My research has concentrated on the following theorem: Any finite covering by affinoid domains of an affinoid space is acyclic. While

this theorem has been proved for one-dimensional spaces, there was known by now no proof for higher-dimensional spaces. I worked out a proof for two-dimensional spaces and am about to generalize the method to the case of arbitrary dimension. This theorem yields a lot of consequences in the theory of non-archimedean functions. I intend to write down these investigations in a joint paper with H. Grauert about affinoid domains.

Author: Jerome A. Goldstein

Abstract evolution equations of the form

$$u''(t) + (A(t) + P(t))u(t) = 0 \qquad (' = d/dt) \qquad (1)$$

for $t \in R = (-\infty, \infty)$ with initial conditions

$$u(0) = f, \; u'(0) = g \qquad (2)$$

are studied. For each t, $A(t)$ is a positive self-adjoint operator on a Hilbert space H, and D = domain $(A(t))$ and W = domain $(A(t)^{1/2})$ are independent of t. Each $P(t)$ is a bounded map from W (equipped with a graph norm) to H. Certain local (in t) Lipschitz conditions are imposed. Then (1), (2) are solved uniquely for $f \in D$, $g \in W$.

As a special case $A(t)$ can be taken to be a formally self-adjoint uniformly elliptic operator of order $2m$ in a (possibly unbounded) domain in R^n. In this case $P(t)$ is an operator of order $\leq m$. Very little smoothness is required of the coefficients of A, P.

There are also some results when the right hand side of (1) is replaced by a non-linear term $M(t,u(t), u'(t))$.

Author: Hervé Jacquet

R. P. Langlands gave a generalization of Weil's result about correspondence

between Dirichlet series and automorphic forms. For that, one needs some facts about the representations of the groups GL(2,K) where K is a local field and in that matter I have been able to improve Langland's proofs and results.

After some more improvements which now avoid tedious and delicate computations, I worked mainly on a generalization of a known fact; although this work is far to be complete, it seems that with two different automorphic forms, one can associate a new family of Dirichlet series with a functional equation; the explicit computation of this equation is rather complicated, but the existence of it is very clear.

Author: Yakar I. Kannai

During this period research was conducted on several questions in the theory of asymptotic behavior of differential problems. In particular, it was discovered that (non-trivial) regular semi-elliptic boundary value problems do exist for domains with curved boundary on manifolds (though not on domains in euclidean spaces) and asymptotic formulas with remainder estimates for the eigenvalues of such problems have been derived. Results were obtained also for some boundary value realizations (such as the Dirichlet problem for semi-elliptic operators) which are not so regular at all points of the boundary. Research on other problems of asymptotic behavior of spectral functions and eigenvalues of differential operators is being carried on at present.

Author: Wilhelm Klingenberg

I continued to investigate the existence and the properties of closed geodesics on compact Riemannian manifolds. As a result, I prepared two papers entitled Closed Geodesics and Simple Closed Geodesics on a Riemannian

Manifold Homeomorphic to the Sphere respectively. A further paper on this subject is in preparation.

Author: Kiyosato Okamoto

Recently M. F. Atiyah and R. Bott proved the Weyl character formula as an application of their result about "a Lefschetz fixed point formula for elliptic complex". I have been researching into the problem of obtaining the character formula for the square-integrable $\bar{\partial}$-cohomology which I had constructed in my recent paper as a generalization of the above-mentioned method.

Author: John V. Ryff

1. Remarks on the monodromy theorem, to be submitted shortly.

2. Paper on the β-topology for the space of bounded holomorphic functions on a region in collaboration with L. Rubel. First draft stage.

3. Paper on three questions of W. A. J. Luxemburg, together with answer to question of mine stated in Muirhead's Theorem. First draft stage.

4. Trying to determine whether bounded automorphic functions always exist with radial limits of modulus 1 almost everywhere. Some progress, but not complete answer as yet.

5. Trying to extend paper of Blumenthal, Phelps and Lindenstrauss on extreme operators in (real) $C(X)$ to complex case. No luck whatsoever. Have example to show that it fails for subalgebras of complex $C(X)$.

Author: Paul J. Sally, Jr.

I concentrated mainly on three problems:

1. Harmonic analysis on the unramified quadratic extension of a local field. This consists of a detailed study of the action of the Fourier trans-

form on certain subspaces of L^2 and leads to a study of certain "Bessel functions".

2. The reduction of the discrete series of representations of SL(2,K), K a local field, when restricted to a compact Cartan subgroup.

3. The explicit computation of the characters of the discrete series on the different Cartan subgroups of SL(2,K).

Author: Shunichi Tanaka

I have been concerned with some irreducible representations of adele groups of SL(2) which can be realized on the space of cusp forms. We obtain a system of such representations parametrized by the character group of $G_k \backslash G_A$, where G is an orthogonal group of two variables. This result was obtained by collaboration with J. Shalika and based on the recent results of A. Weil (Acta 111).

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13. ABSTRACT

The unifying theme in the Advanced Mathematical Research supported by the grant during the past year is analysis, as it has been in past years. During this year some 16 manuscripts were submitted by the mathematicians supported by the grant and some more are in preparation. The titles of these manuscripts indicate, and the research summaries in the Addendum attached to this report give further evidence of the considerable diversity of the research undertaken. While some of this work belongs to the central or even classical parts of analysis (Kannai, Goldstein, Ryff), most of it deals with areas where analysis interacts with other disciplines. Some of these contact areas are rather new, as exemplified by the work of Atiyah combining algebraic topology with the study of elliptic operators or, in another direction, by the work of P. Sally, where new connections with number theory appear.

Considering the broad spectrum of the research, as well as the significance and depth of some of the problems on which progress was made, the work supported by the grant seems to have been quite successful.

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